THE MARKOVOPOLY PROJECT

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Introduction

As a favorite game amongst many, the game of monopoly is very strategic. Players rotate around the board ultimately bankrupting their opponents through the use of buying and selling property. Our objective is to analyze the game of monopoly by treating it as a Markov chain. We explain our methods of solving this multi-faceted task.

Rules

Jail. The most influential space in the game of monopoly is the Jail space. In fact, players have an edge if they are able to stay in Jail the longest towards the end of the game. It is also considered a safe space where nothing happens but you at least forfeit a turn.

Visiting Jail. There is no penalty incurred in this move.

Going to Jail. There are a few ways this action can be performed:

(a) Your piece lands on 'Go to Jail'.
(b) Your piece lands on a 'Community Chest' or 'Chance' card that directs you to this position.
(c) Your misfortune of rolling three consecutive doubles.

The Jail Release. The early release possibilities:

(a) The act of rolling doubles while in Jail on any of the (3) turns.
(b) Using the Get out of Jail Free Card.
(c) Paying a $50 fine before rolling the dice for the first or second turns.

Additional Ways of Movement.

(a) *Chance:* There are 9 additional ways to redirect how you move.
(b) *Community Chest:* There are 2 ways to redirect your movement.

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Property. The winning objective is to make all of the other players bankrupt in the game of monopoly. There are 3 ways to purchase property:

(1) Landing on the property space and purchasing it.
(2) Being the highest bidder in an auction for property.
(3) Buying it from an opponent in a trade.

Note : Properties may be received from bankrupt players assuming the bank wasn't responsible.
Note : To acquire a monopoly a player must own all of the properties in a color group.

Dice. In the game of monopoly two dice are rolled simultaneously. As a result that indicates the number of moves allotted that turn. The dice are called **doubles** if they share the same number when rolled. More than two consecutive doubles will land that player in jail.

The code

We use a block matrix (D) to represent just rolling the dice. We call this our *Rolling Matrix*.

$$D = \left[\begin{array}{c|c} M_1 & M_2 \\ \hline M_3 & M_4 \end{array}\right]$$

We let M_1 = traveling around the square. (120×120)
We let M_2 = leaving Jail by rolling doubles or by buying out. (120×3)
We let M_3 = Roll doubles three times in a row and go to Jail. (3×120)
We let M_4 = Staying in Jail. (3×3)

We use the *Repositioning matrix* for when the game forces you to move rather than the dice. We have a matrix C that is a 123 by 123 approximate identity matrix that moves the player with using the Chance cards, Community Chest and Go to Jail cards.

One major strategy for playing Monopoly pertains to a player's choice of how to react to being in Jail. When a player is in Jail, they can still collect rent from other players who land on their properties, but they may not buy new properties. Towards the beginning of the game, when there are more properties available to buy, it is to the player's advantage to buy their way out of Jail as soon as possible. However toward the end of the game, the player should try to stay in Jail for as long as possible. We developed different matrices C and D for both strategies to model both the late game probabilities and the early game probabilities.

The Markovopoly Analysis

To determine the limiting probabilities for the game, we first multiply matrix C (the *repositioning* matrix) with matrix D (the *rolling* matrix) to get a matrix M. Our two matrices M (from the early game and the late game) can then be raised to a sufficiently high power to find the limiting probabilities for each case of the game.

The limiting probabilities for early game. The Square in Order of Decreasing Probability:

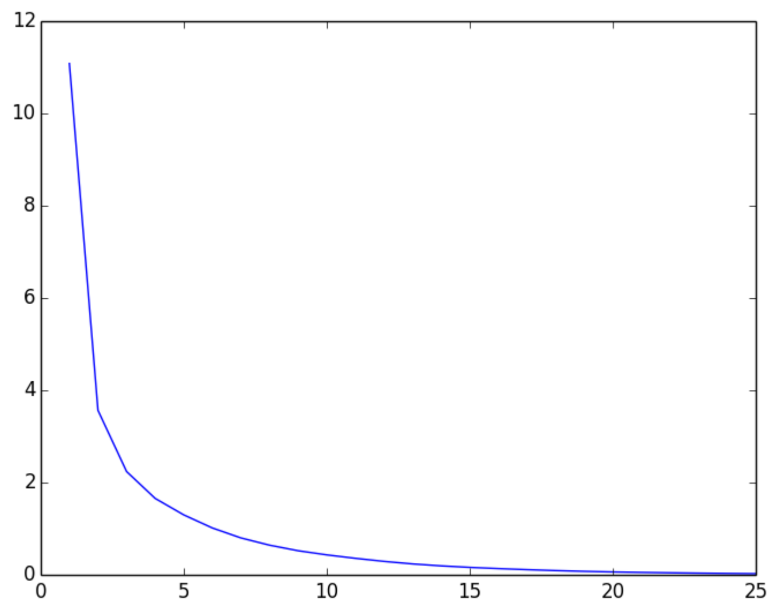
Just Visiting: 0.0588918334626
Illinois Ave: 0.0319845400559
Go: 0.0316592161408
New York Ave: 0.030958912263
B&O Railroad: 0.0305281740953
Reading Railroad: 0.0298596974622
Pennsylvania Railroad: 0.0292752362394
Tennessee Ave: 0.029051789134
Free Parking: 0.028463686441
Kentucky Ave: 0.0283548009918
Water Works: 0.0282415293307
St. James Place: 0.0276293536444
St. Charles Place: 0.0274912084126
Indiana Ave: 0.0274467687798
Atlantic Ave: 0.0271294692174
North Carolina Ave: 0.0268603203038
Pacific Ave: 0.0266859446603
Ventnor Ave: 0.026623020752
Electric Company: 0.0261201000792
Boardwalk: 0.0259972235309
Community Chest 2: 0.0258288353125
Marvin Gardens: 0.0257646838954
Pennsylvania Ave: 0.0256376599354
Virginia Ave: 0.0245880363613
States Ave: 0.0240089639463

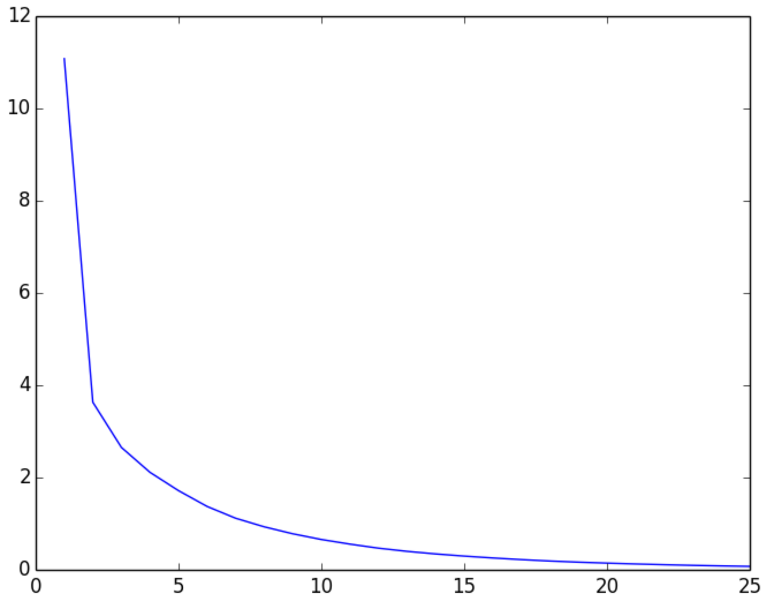
Short Line Railroad: 0.0239325084723
Vermont Ave: 0.0236972463079
Income Tax: 0.023

The limiting probabilities for late game. The Square in Order of Decreasing Probability:

Jail: 0.084753695433
Just Visiting: 0.0406035313357
Illinois Ave: 0.0296338724054
Go: 0.0293752579249
B&O Railroad: 0.0283742303022
Reading Railroad: 0.0278553105148
Free Parking: 0.0277956861699
Tennessee Ave: 0.0276660323281
New York Ave: 0.0275329457049
Water Works: 0.0263090289333
St. James Place: 0.0262653849778
Electric Company: 0.0262476330668
Pennsylvania Railroad: 0.0257362391542
St. Charles Place: 0.0256751044348
Kentucky Ave: 0.0255788896925
Indiana Ave: 0.0253234892877
Atlantic Ave: 0.0250473939803
North Carolina Ave: 0.0249208423961
Pacific Ave: 0.0248199936133
Ventnor Ave: 0.0246548623288
Boardwalk: 0.0242807259007
Virginia Ave: 0.0241109169021
Marvin Gardens: 0.0239564094819
Pennsylvania Ave: 0.0237611683023
Short Line Railroad: 0.0222467992332
Community Chest 2: 0.0221080942221
Vermont Ave: 0.0220289120294
Income Tax: 0.02198165

The rate of Convergence to the limiting distribution. To discuss the rate of convergence of our Markov process, it is sufficient to look at the norms of the eigenvalues. If they are sufficiently small, then we should expect our Markov process to converge to the steady state rather quickly. If they are close to 1, however, we should expect slower convergence. Unfortunately, we get floating point errors when we attempt to use numpy.eig to compute the eigenvalues, so we turn to a different method. To find the steady state distribution, we use the power method; i.e., we raise our matrix M to the $100,000,000$ power and look at the first column. To find the rate of convergence, we graph the distance between M^n and $M^{100,000,000}$ as a function of n (using the metric induced by the 2-norm). As the graph demonstrates, by $n = 25$, the matrix is almost indistinguishable from the steady state.





The expected income per turn of properties and groups of properties. To determine the expected income per turn of properties, we use the probabilities given above as well as the rent that would be paid to the property owner (assuming that they do not have a monopoly). By multiplying the probability of landing on a property with the rent gained by the property owner, we can rank properties by the initial expected income.

When a player owns all of the properties in a given color set, the rent paid by visiting players on each property is doubled. This is sometimes referred to as having a monopoly over that area of the board. By taking the sum of the expected incomes of the properties in the color set, we can rank color sets from most lucrative to least lucrative. Some color sets include three properties, while others have only two, so we also include the expected return per property in a color set.

Most lucrative monopolies: Probability, Expected Return, Expected Return/Square:

Red:

$$\begin{array}{r} 0.0877861098275 \\ 4.22011473451 \\ 1.4067049115 \end{array}$$

Orange:

$$\begin{array}{r} 0.0876400550414 \\ 4.12025470252 \\ 1.37341823417 \end{array}$$

Yellow:

$$\begin{array}{r} 0.0795171738649 \\ 3.60181438564 \\ 1.20060479521 \end{array}$$

Green:

$$\begin{array}{r} 0.0791839248994 \\ 3.28823811401 \\ 1.09607937134 \end{array}$$

Pink:

$$\begin{array}{r} 0.0760882087203 \\ 2.57775719021 \\ 0.859252396737 \end{array}$$

Light Blue:

$$\begin{array}{r} 0.0699332302376 \\ 1.62011631985 \\ 0.540038773284 \end{array}$$

Dark Blue:

$$\begin{array}{r} 0.047719114237 \\ 0.932237554776 \\ 0.466118777388 \end{array}$$

Brown:

$$\begin{array}{r} 0.0424172521388 \\ 0.255357656125 \\ 0.127678828062 \end{array}$$

Most lucrative monopolies: Probability, Expected Return, Expected Return/Square:

Orange:

$$0.0814643630107$$
$$3.91714889741$$
$$1.30571629914$$

Red:

$$0.0805362513856$$
$$3.84204255247$$
$$1.28068085082$$

Yellow:

$$0.073658665791$$
$$3.33680693273$$
$$1.11226897758$$

Green:

$$0.0735020043116$$
$$3.0178405395$$
$$1.0059468465$$

Pink:

$$0.0713306216318$$
$$2.39113394712$$
$$0.79704464904$$

Light Blue:

$$0.0650107880199$$
$$1.52305610024$$
$$0.507685366748$$

Dark Blue:

$$0.0444802967921$$
$$0.866617560837$$
$$0.433308780418$$

Brown:

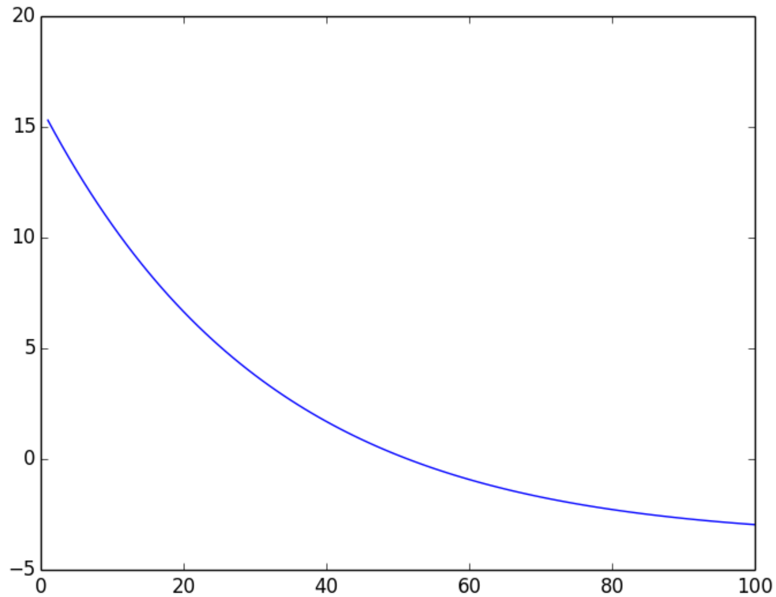
$$\begin{array}{r} 0.0394075452705 \\ 0.237239714454 \\ 0.118619857227 \end{array}$$

The best point in the game to switch from staying in jail optimally. To determine the optimal time to switch from the early game strategy of immediately paying 50 to get out of jail to the late game strategy of staying in jail until you roll doubles or by serving a three turn sentence, we attempt to plot the expected future gain as a function of time. First, we calculate the average gain from landing on each of the three chance and community chest spaces, as well as the loss from landing on Luxury Tax or Income Tax. Multiplying those values by the probabilities of occupying those squares, we get the expected per turn gain from landing on bank-owned properties. Next, we determine the probability of landing on an unowned square as a function of time.

$$p(n) = \frac{28}{40} \cdot \left(\frac{39}{40}\right)^{n-1}.$$

The rent you can expect to secure on a given turn is this probability times the rent times the number of remaining turns of the game times the proportion of other players to total players. We also calculate the average amount of rent you pay by multiplying the probability of landing on an owned property times the average rent. Summing these values we associate a value to each turn of the game, and the point that this value becomes negative is when the player should switch to the late game strategy. Testing this for several estimates of game length, we find that this point occurs about 30% through each game.

The best place from buying out of Jail to staying in Jail.



The property in a group be the first to get a house. After obtaining a monopoly over a color set, the player can choose to develop the property with houses and hotels. After purchasing a house for one property in a color set, houses must be purchased for the other properties in the color set before a second house may be purchased for the initial property. Since the probability that a player will land on a given property in a color set differs depending on the color set, and the rent of different properties in a color set are often different, we can combine these factors to develop a ranking for how to allocate development resources. For each color group, going clockwise around the board, the last property will bring in the most rent, and therefore should be the property for which houses are purchased first. Depending on the color set, the second property to buy a house for would be either the first or second property.

Properties that should be first to get a house.

Brown:

$$Property2 : 0.512423598833$$
$$Property1 : 0.381328197603$$

Light Blue:

$$Property3 : 1.49047030633$$
$$Property2 : 1.35878798832$$
$$Property1 : 1.34581070185$$

Pink:

$$Property3 : 2.50528562886$$
$$Property1 : 2.44485257223$$
$$Property2 : 2.34038523824$$

Orange:

$$Property3 : 4.06378497884$$
$$Property2 : 3.79793233384$$
$$Property1 : 3.73819004328$$

Red:

$$Property3 : 5.20731051737$$
$$Property1 : 4.81939736757$$
$$Property2 : 4.77036362812$$

Yellow:

$$Property3 : 5.4568716261$$
$$Property1 : 5.39235935399$$
$$Property2 : 5.35893375527$$

Green:

$$Property3 : 6.63005476844$$
$$Property2 : 6.31521971821$$
$$Property1 : 6.30161841801$$

Dark Blue:

$$Property2 : 6.7199770556$$
$$Property1 : 6.40105322666$$