Distributed Frequency Regulation in Islanded Microgrids

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1 Introduction

A set of power generators and loads is considered a microgrid if, in comparison to larger power systems, they are smaller and have lower ratings and capacities. An islanded microgrid is a microgrid that is not connected to a larger power system. Generally, ac power systems have three main objections. First, they must always balance power generation with demand. Secondly, they must regulate frequency. Lastly, they should optimize cost. The project I am working on deals primarily with the second objective: regulating frequency.

Large power systems that are already in place typically use a centralized controller to perform this objective; information is sent between the power generators and centralized controller that helps to regulate the frequency. Recent work has proposed distributed approaches to frequency regulation in microgrids [1–4]. Most of these approaches use an undirected communication network between the generators. What I have done is study different distributed approaches to regulating frequency in islanded microgrids that use directed communication networks between generators. I have programmed a simulation to see how the communication network affects frequency regulation in the microgrid and made some observations based on data from this simulation.

2 Model

The model for islanded microgrids used is an electrical network with n generators and l loads. The dynamics of the n generators were described by a constant voltage behind reactance model, augmented to include a frequency-droop controller [4]. The equations governing this are:

$$\frac{d\theta_i}{dt} = \omega_i - \omega_0 \quad (1)$$

$$D_i \frac{d\theta_i}{dt} = u_i - \sum_{j=1}^{n+l} V_i V_j B_{ij} \sin(\theta_i - \theta_j) \quad (2)$$

where ω_0 is some nominal system electrical frequency, θ_i is the phase angle of the voltage of the i^{th} component measured with respect to a reference frame rotating at ω_0, D_i is the speed-droop characteristic slope of the power supply, u_i is the the generation set-point of the i^{th} generator, V_i is the voltage of the i^{th} component, and B_{ij} is the imaginary part of the admittance from node i to node j.

The loads were assumed to be constant power loads. In this case, they must satisfy:

$$D_i \frac{d\theta_i}{dt} = -P_i^d - \sum_{j=1}^{n+l} V_i V_j B_{ij} \sin(\theta_i - \theta_j) \quad (3)$$

where P_i^d is the power demand of load i and all other variables are as before.

Notice, when all the components are operating synchronously, by adding equations (2) and (3) for all i, we have:

$$\sum_{i=1}^{n+l} D_i \frac{d\theta}{dt} = \sum_{i=1}^{n} u_i - \sum_{i=n+1}^{n+l} P_i^d$$

Thus, we have:

$$\Delta\omega = \frac{d\theta}{dt} = \frac{\sum_{i=1}^{n} u_i - \sum_{i=n+1}^{n+l} P_i^d}{\sum_{i=1}^{n+l} D_i}$$

The goal in frequency regulation is to vary u_i in such a way so that $\Delta\omega$ goes to 0.

3 Simulation

The program that I wrote in Matlab to simulate a microgrid with n generators and l loads and the aforementioned dynamics takes in three inputs. The first input is an $(n + l) \times (n + l)$ matrix. It is the adjacency matrix of the graph describing the electrical network. The second input an $n \times n$ matrix. It is the adjacency matrix of the directed graph describing the communication network between the nodes. The third input is a length $n+l$ row vector that is $< \theta_i(0) >$. The values used for the other parameters can be found in Table 1.

Parameter	Symbol	Value
Nominal Voltages	V_i	120 V
Speed-droop Coefficients for Generators	D_i $\quad i \leq n$	5 kW $\cdot$ s
Speed-droop Coefficients for Loads	D_i $\quad i \geq n+1$	0.5 kW $\cdot$ s
Admittance	B_{ij} $\quad (ij \in E)$	$\frac{1}{6\pi} \cdot 10^{-2}$ S [1]
Load Size	P_i^d	3 kW
Feedback Constant	α	Varied

Table 1: Parameter values for the simulation

The program then uses ode23s to solve the system of differential equations described by (2) and (3). The program then checks to see how long it took for the system to stabilize and have a frequency error of less than 1 rad/sec. It did this by simply running a loop over time and at each instance checking if the range of $< \frac{d\theta_i(t)}{dt} >$ was less than 0.1. If it was, it would check if $\Delta\omega$ was less than 1. To verify that this was not just a coincidence, the program makes sure that these conditions are held by the system for 1 continuous second. The function then returns the time at which the system has been stable for 1 second.

4 One Approach to Frequency Regulation

One possible idea for frequency regulation that my advisor had me study was with respect to the small case $n = l = 1$. The feedback was given by $u_i = \alpha(\theta_1 - \theta_2)$ for some value α. In this case it is possible to reduce the two-dimensional system to a one-dimensional system, making it easier to analyze. By manipulating equations 2 and 3 with $i = 1$ and $i = 2$, respectively, one obtains:

$$\frac{d\phi}{dt} = -\frac{\alpha\phi}{D_1} + \frac{P_2^d}{D_2} - \alpha(\frac{1}{D_1} + \frac{1}{D_2})\sin(\phi)$$

where $\phi = \theta_1 - \theta_2$. Any equilibria of the two-dimensional system will also be equilibria of the one-dimensional system. For simplicity, I will refer to the system as:

$$\frac{d\phi}{dt} = -m\phi + b - A\sin(\phi)$$

for some constants m, b, and A. The equilibria points of this system occur when

$$A\sin(\phi) = -m\phi + b$$

Depending on the constants A, m, and b, this equation can have any number of solutions. In fact, if $\frac{m}{A} = |\cos(x_n)|$ where x_n is the n^{th} non-negative solution

[1]This value of admittance corresponds to a nominal frequency of 60 Hz and a line inductance of 0.5 mH

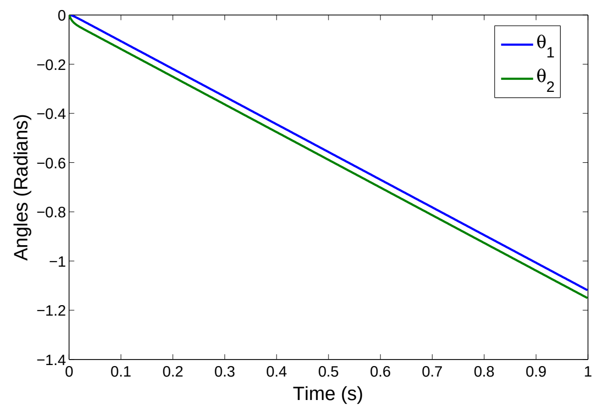


Figure 1: One instance of the simulation where $\frac{d\theta}{dt}$ is not driven towards zero

to $tan(x) = x$, then the system has $2n - 1$ solutions. However, at most one of these equilibria can be an equilibrium point of the two-dimensional system. This is because if (θ_1, θ_2) is an equilibrium point of the two-dimensional system, then we have:

$$0 = -\alpha(\theta_1 - \theta_2) - V_1 V_2 B_{12} \sin(\theta_1 - \theta_2)$$
$$0 = P_2^d - V_1 V_2 B_{21} \sin(\theta_2 - \theta_1)$$

Adding these two equations and using the fact that $B_{12} = B_{21}$, we obtain

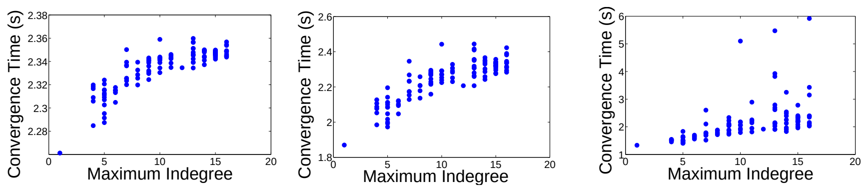
$$\alpha\phi = -P_2^d$$

From this it is easy to see that at most one of the ϕ that is an equilibria point of the one-dimensional system corresponds to an equilibria in the two-dimensional system. The other such ϕ have $\frac{d\theta_1}{dt} = \frac{d\theta_2}{dt} \neq 0$. Thus, for these ϕ, $\frac{d\theta_i}{dt}$ is not driven to zero.

In the simulation, it was possible to find a case that exhibited such behavior. Using the system described above with $\theta_1(0) = 0, \theta_2(0) = 0$, and $\alpha = 100{,}000$, one sees the results shown in Figure 1. Thus, this method of feedback does not seem to be desirable in frequency regulation.

5 A Second Approach to Frequency Regulation

Another idea my advisor had me explore was used not just on a system with $n = 1, l = 1$, but on larger systems as well. In this approach, u_i was set to be:



(a) $\alpha = 1,000$ (b) $\alpha = 10,000$ (c) $\alpha = 100,000$

Figure 2: Maximum indegree of the communication network against maximum convergence time for varying values of α. The maximum convergence time, seen as a worst case scenario for convergence time, is taken across 30 different sets of initial conditions.

$$\frac{\sum_{j\in\mathcal{N}_i^-} D_j\theta_j}{\sum_{j\in\mathcal{N}_i^-} D_j}$$

where $\mathcal{N}_i^-$ is the in-neighborhood of node i in the communication network. As a convention, each node was always taken to be in the in neighborhood of itself.

My advisor was able to show that in the case that the communication network is a complete digraph on n nodes, the frequency error is always driven towards zero. My job was to use my simulation to check what happens when edges are removed from the communication network, both in terms of does the system still drive the frequency error to zero and how long does this process take.

In all of the networks for which the simulation was run, the frequency error was driven to zero. Through running this simulation with varying input parameters, I realized that, for a fixed electrical system and set of initial conditions, varying the communication network had little affect when the electrical system was well connected. For this reason, I then focused on electrical systems whose corresponding graphs had higher diameters. The majority of the data was collected with $n = 16, l = 9$ and diameters at least 8.

In looking at a few different electrical systems, I believe that the frequency regulation occurs faster when the network graph is less connected. Which network graph parameter affects convergence rate the most, I do not know. Figure 2 shows the worst case convergence time plotted against the maximum indegree of the communication network for various values of α. This figure is in reference to a fixed electrical system, though a similar analysis was done for a few systems.

6 Conclusion

The first feedback system that I studied, where $u_1 = \alpha(\theta_1 - \theta_2)$, is not a valid way to regulate frequency. This was seen both in an analysis of the system and in simulations of the system.

The second feedback system that I studied does seem to be a valid way to regulate frequency. Because the size of the system was much larger, only simulations were used to study the system. In this study, the frequency error was always driven to zero. In addition, the worst case convergence time was faster when the communication network was less connected. The fastest time was observed when the communication network simply consisted of each generator only giving information to itself.

References

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